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THE INFLUENCE OF A VARIABLE DEPTH ON STEADY ZONAL BAROTROPIC FLOW

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THE INFLUENCE OF A VARIABLE DEPTH ON STEADY ZONAL BAROTROPIC FLOW

by

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TABLE OF CONTENTS

Chapter

I INTRODUCTION 1

I-1 Oceanographic Investigations 2

I-2 Meteorological Investigations 6

I-3 Initial Evaluation of the Problem 7

II FORMULATION OF THE PROBLEM 9

II-1 Basic Equations 9

II-2 The Vorticity Equation 9

II-3 The Transport Streamfunction 10

III SOLUTION FOR ZONAL FLOWS 12

III-1 Simplification of the Problem 12

III-2 Matching Conditions 13

III-3 The General Solution 13

IV EVALUATION OF THE SOLUTION FOR SOME SIMPLE PROFILES 16

IV-1 The Bottom Profile is a Simple Step 16

IV-2 The Bottom Profile Resembles a Plateau 17

IV-3 Other Discontinuous Bottom Profiles 20

IV-4 The Bottom Profile is Smoothly Sloping 21

V DISCUSSION 23

V-1 Physical Interpretation of Results 23

V-2 Comparison with Previous Investigations 26

V-3 Numerical Evaluation and Comparison 27

(a) Atmospheric Example 27

(b) Gulf Stream Meanders 28

Chapter

(c) South Atlantic 31

(d) Antarctic Circumpolar Current 32

V-4 Comparison and Discussion of Amplitudes 33

VI SUMMARY 36

BIBLIOGRAPHY 37

FIGURES 40

LIST OF FIGURES

Figure

1. Transport lines around the Antarctic Continent 41
2. Dynamic topography of the South Atlantic 42
3. Zonal flow over a shoaling step 43
4. Zonal flow over a narrow plateau 44
5. Zonal flow over a wide plateau 45
6. Unstable flow over a plateau 46
7. Zonal flow over a plateau (width: 1 wavelength) 47
8. Zonal flow over an idealized valley 48
9. Zonal flow over a transition slope 49

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ABSTRACT

The barotropic vorticity equation is utilized to investigate the steady topographic disturbance of an initially uniform zonal current. Solutions are obtained on the beta-plane for various discontinuous bathymetric profiles. It is shown that a smooth slope does not appreciably modify the results. Stability criteria are derived that define the values of the flow parameters for which the streamlines become discontinuous.

The solutions are evaluated for several observed disturbances of oceanic currents. The velocities implied by the observed wavelengths are physically realistic, but the discrepancy between observed and calculated amplitudes suggests that the vertical structure of the velocity profile and the baroclinicity of the ocean are important.

CHAPTER I

INTRODUCTION

In most studies of the large scale circulation patterns of the oceans, the sea floor is assumed to be either flat or infinitely deep. Such simplifications are made to facilitate the investigation of certain classes of flows in which it is suspected that topographic effects are nonexistent or of minor importance. That currents in the real ocean are to some extent topographically affected has been pointed out by many authors. Perhaps the most universally cited example of such an effect is the wave-like structure in the dynamic topography of the Antarctic Circumpolar Current (ACC) where it crosses each of several ridges (see Figure 1).

Many contributions to the understanding of oceanic currents have been achieved through the application of the techniques and principles of dynamic meteorology. That many apparent similarities exist between oceanic and atmospheric flows was graphically demonstrated in 1951 when Fuglister and Worthington published the results of an intensive multiple ship survey of a Gulf Stream meander. Their illustration of the motion and incipient occlusion of the meander bears a striking resemblance to an earlier illustration (Berggren, _et_ _al_., 1949) of the unstable waves observed at the 500 mb level in the atmosphere. It will therefore be found instructive to consider briefly the results of previous investigations of topographic effects on both oceanic and atmospheric flows.

I-1. Oceanographic Investigations

Ekman (1923), in an attempt to account for the deflection of the Gulf Stream over the Southeast Newfoundland Ridge, considered the effect of bathymetric relief on a homogeneous current under a balance of pressure gradient, bottom friction, and constant Coriolis forces. His results show the current deflection to be dependent on the slope of the bottom, but not the total depth, and give a net lateral displacement of the streamlines (to the right for flow over a ridge in the Northern Hemisphere). For flow over successive ridges, Ekman's streamlines are 90 degrees out of phase with the topography, the maximum deflection occurring at the point of inflection of the bottom profile.

In 1932 Ekman reconsidered the problem and included accelerations in addition to bottom friction, constant Coriolis force, and pressure gradients. Subject to the requirements that both the bottom slope and the variation in depth be small, he obtained a solution that gives streamlines in phase with the bathymetry, the maximum displacement to the right occurring over the point of minimum depth.

In an attempt to rigorously investigate the effect of bottom friction on a topographically disturbed homogeneous current, Görtler (1941) utilized Prandtl's boundary layer theory. He simplified the problem by considering a constant Coriolis parameter and a variation in depth that is small compared to the total depth of the fluid. His solution to the linearized equations reduces to that of Ekman (1932) provided that Görtler's friction parameters, including the boundary layer thickness, are properly identified with Ekman's "depth of frictional resistance." Görtler extended Ekman's solution by including the possibility of the upstream flow approaching the topographic feature at other than normal incidence. He also

evaluated the frictional effects for several physically reasonable values of the flow parameters and thereby demonstrated that such effects are negligible for most atmospheric and oceanic currents. Görtler pointed out though, as did Ekman (1932), that topographic effects in which inertia plays an important role may not properly be treated as independent of planetary effects when the scale of motion is such that the latter must be considered. This point is central to the investigation at hand.

Sverdrup (1941) took a new approach to the problem of topographic effects. He neglected friction and considered that the effect of topographic features was to disturb the internal distribution of mass. Being unable to treat the problem rigorously, he nevertheless was able to demonstrate qualitatively that a bathymetric feature which rises from a layer of no motion into an internal solenoidal field and its associated current will disturb that current in such a way that the physical sea surface will be depressed above the feature. This effect causes the current to deviate to the right (in the Northern Hemisphere), the maximum deviation being over the point of minimum depth, and then to return to its original configuration downstream. That the converse is true for flow over a depression is not clear, as there is no mechanism for disturbing the solenoidal field. Sverdrup's conclusions agree qualitatively with the observed features of the circulation in the Southern Ocean.

Shtokman (1947), in a discussion of his subsequently published (1948b) theory of topographic effects on ocean currents, demonstrated an apparent serious discrepancy between the theories of Ekman, Görtler, and Sverdrup and observed flow features of the South Atlantic and Caspian Sea. Considering Defant's (1941) chart of the "absolute" topography of the surface in the Atlantic, and relating this chart to the bathymetry of the South Atlantic

between 25° and 50° South, Shtokman observed that the eastward currents are deflected poleward over shoaling water and equatorward over deepening water. This is contrary to the observed deflection of the ACC. Since the direction of deflection in the earlier theories depended directly on the sign of the Coriolis parameter, there was no mechanism available to explain opposite deflections in the same hemisphere.

Noting the above apparent discrepancy between theory and observation, Shtokman inferred that the Coriolis parameter, if considered constant, should not be the primary consideration. On this basis, he adapted an earlier (1948a) derivation of the equations relating wind stress to lateral friction to the problem of a variable depth. For zero wind stress, the signs of the terms in his equations give qualitative agreement with Sverdrup's (1941) conclusions. Shtokman's approximation of the complete equations including wind stress implies that the direction of the current deflection over a topographic feature depends on the sign of the local curl of the wind stress and not on the hemisphere. Thus, for shoaling water, a negative wind stress curl yields a deflection to the right, and vice versa.

As Shtokman pointed out, his results agree with the observations he cited. Thus the absence of perturbations on Defant's 1941 topography of the surface of the South Atlantic between 40° and 50° South is explained by the existence of maximum wind stress, and hence zero wind stress curl, in these latitudes. A later presentation by Defant (1961) of the dynamic topography of the same area indicates substantially the same structure except for the addition of an anticyclonic eddy at approximately 45°S - 20°E (see Figure 2). If this is indeed an area of zero wind stress curl, then Shtokman's theory does not account for this eddy.

The writer believes that a more fundamental objection to Shtokman's theory may be made on the basis of his method of introduction of a variable depth. The depth which Shtokman ultimately considers as representing the actual bathymetry was originally defined as that depth at which the transport components vanish. Charney (1955) and Fofonoff (1962) have shown that this depth of no motion is a function of the transport itself, and hence is of a nature that is fundamentally different from the bathymetry.

Shtokman's refutation of the theories of Ekman (1923), Görtler (1941), and Sverdrup (1941) based upon consideration of the near shore currents in the Caspian Sea seems inappropriate in that those earlier theories specifically excluded changes of depth that amount to a significant fraction of the total depth. The earlier theories also excluded flows initially parallel to the isobaths, such as are considered by Shtokman (1947, Fig. 5).

Neumann (1960) attempted to combine analytically the results of Ekman (1923) and Sverdrup (1941). Neumann initially considered the steady flow in a baroclinic ocean to be maintained by a balance of Coriolis, friction (both wind stress and "internal" friction), and pressure gradient forces. He pointed out, as have Defant (1941) and others, that for homogeneous water, the variation of Coriolis parameter with latitude requires that all purely geostrophic currents be zonal, unless the depth increases from the equator toward the poles in proportion to the sine of the latitude. Neumann obtained approximate solutions for several classes of flow, including that of homogeneous water flowing over a ridge under the influence of a variable Coriolis parameter. This latter solution gives a Sverdrup-type deflection of the streamlines without the consideration of a solenoidal field. Neumann's solutions are based upon the requirement that the change

of depth be small compared to the total depth, but the numerical examples given do not clearly meet that criterion.

Saint-Guily (1962) has extended his earlier (1959) treatment of the generalized Ekman problem to the case of a variable depth. Subject to the assumptions of vanishing friction and moderate changes in depth, Saint-Guily obtained solutions for the steady zonal flow of homogeneous water over a sill that correspond to those of Neumann (1960) for a variable Coriolis parameter. Saint-Guily's initial disregard of the field acceleration terms in the equations of motion restricts the validity of his solutions to regions where the ratio of the relative vorticity to the Coriolis parameter (Rossby Number) is small.

I-2. Meteorological Investigations

As was noted above, topographic effects on ocean currents have parallels in meteorological problems. Queney (1949) summarized previous theoretical treatments of topographic effects, including the results of his own extensive treatment of the problem (1947) utilizing perturbation techniques. Queney considered the stability associated with the density stratification of an infinitely deep atmosphere, and it is therefore difficult to apply his results directly to a homogeneous ocean. Charney and Eliassen (1949) concluded that the real gravitationally stable atmosphere may be treated as an "equivalent barotropic atmosphere" whose properties coincide with those of the real atmosphere at a height of zero horizontal divergence (approximately 500-700 mb). Charney and Eliassen were able to substantially improve their numerical prediction techniques by including a rough model of the effects of continental topography. The importance of such continental effects in the study of large scale atmospheric flows has been discussed by Bolin (1950). He noted the regular perturbations of the

pressure surfaces that occur on monthly (and longer) mean weather charts of both the Northern and Southern Hemispheres and argued in favor of a topographic mechanism for the generation of such perturbations as opposed to a thermodynamic mechanism. Bolin linearized the equations and obtained approximate solutions for flow over an isolated mountain.

Oi (1956) considered the problem of the frictionless barotropic flow of air over a mountain range under the influence of a constant Coriolis parameter. He was able to include accelerations in a manner similar to Ekman (1932) and, again for small changes in the total depth of the fluid column, obtained approximate solutions showing the effect of a mountain range on an initially zonal flow.

I-3. Initial Evaluation of the Problem

All of the above investigations, with the exception of that of Shtokman, have essentially linearized the problem by considering only small changes in relief relative to the total depth of the fluid column. This may be shown (Bolin, 1950) to be equivalent to assuming that the relative vorticity of the fluid is small compared to the local value of the Coriolis parameter. In the ocean, with a mean depth of about 4 km, and in the equivalent homogeneous atmosphere, with a height of 8 km, it is apparent that there are many areas where such a linearization will be inappropriate. At low latitudes the relative vorticity may certainly approach or exceed the value of the Coriolis parameter. In the oceans there are innumerable submarine features, covering a spectrum of size scales, that rise well above the mean surrounding depths. These observations on the inherent limitations of the perturbations techniques, coupled with the well known importance of the variation of the Coriolis parameter with latitude, provide the basis for the present investigation.

In subsequent chapters, Görtler's (1941) conclusions will be accepted and it will be assumed that the flow of the barotropic ocean is frictionless, although it is recognized that friction must be important in some limiting sense. Indeed, Görtler pointed out that frictional effects may become important when the horizontal scale of the topographic feature under consideration is very much larger than the inertial radius of the flow. As the objective of this investigation is to study the combined effects of the variation of Coriolis parameter and finite changes in depth over a range of scales, subsequent assumptions will not restrict these essential features of the problem.

The concept of vorticity has become fundamental to the investigation of both atmospheric and oceanic flows. In its general form, the vorticity equation is a concise statement of one of the laws of fluid motion. For the class of flow under consideration its form facilitates investigation from either an Eulerian or Lagrangian viewpoint. This property of the vorticity equation will be utilized in succeeding chapters.

The deep currents in the oceans will be treated as though they were barotropic, not because this mode is necessarily strictly representative of in situ deep currents, but because the barotropic equations of motion are simpler to treat exactly than are the baroclinic equations. The influence on this barotropic flow of topographic features with the scales of features existing in the real ocean will be investigated.

Topographic effects observable in the baroclinic flow (see Figure 1) must be related to those occurring in the barotropic flow, but this relation has not yet been determined.

CHAPTER II

FORMULATION OF THE PROBLEM

II-1. Basic Equations

We consider the horizontal components of the equation of motion of a frictionless, homogeneous ocean or atmosphere,

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial P}{\partial x}, \qquad \frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial P}{\partial y}, \tag{1}$$

together with the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}, \tag{2}$$

where u, v, and w are the velocity components along the x (eastward), y (poleward), and z (upward) axes. The Coriolis parameter (equal to twice the local vertical component of the earth's rotation) is denoted by f, and P is the local pressure. The local acceleration of gravity is g and ρ is the fluid density, both of which are assumed constant.

II-2. The Vorticity Equation

The two components of equation (1) are cross-differentiated with respect to y and x respectively, and then subtracted to give, after rearrangement of terms,

$$\frac{D(\zeta + f)}{Dt} = -(\zeta + f)\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] + \left[\frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z}\right], \tag{3}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, the vertical component of relative vorticity.

Equation (3) may be considered as the primitive vorticity equation, where the change in the absolute vertical vorticity of a parcel of fluid is

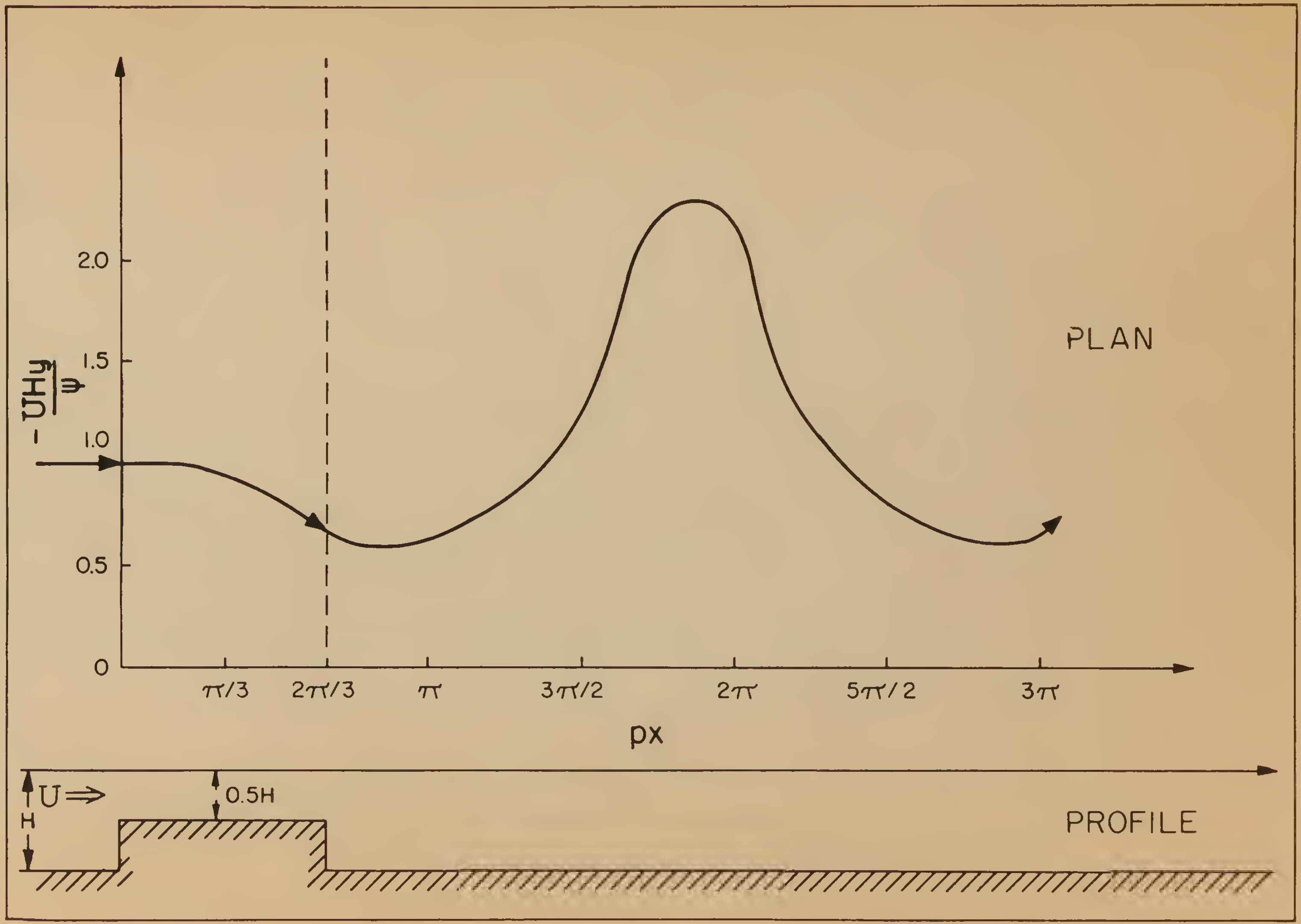

Figure 4. Zonal flow over a narrow plateau

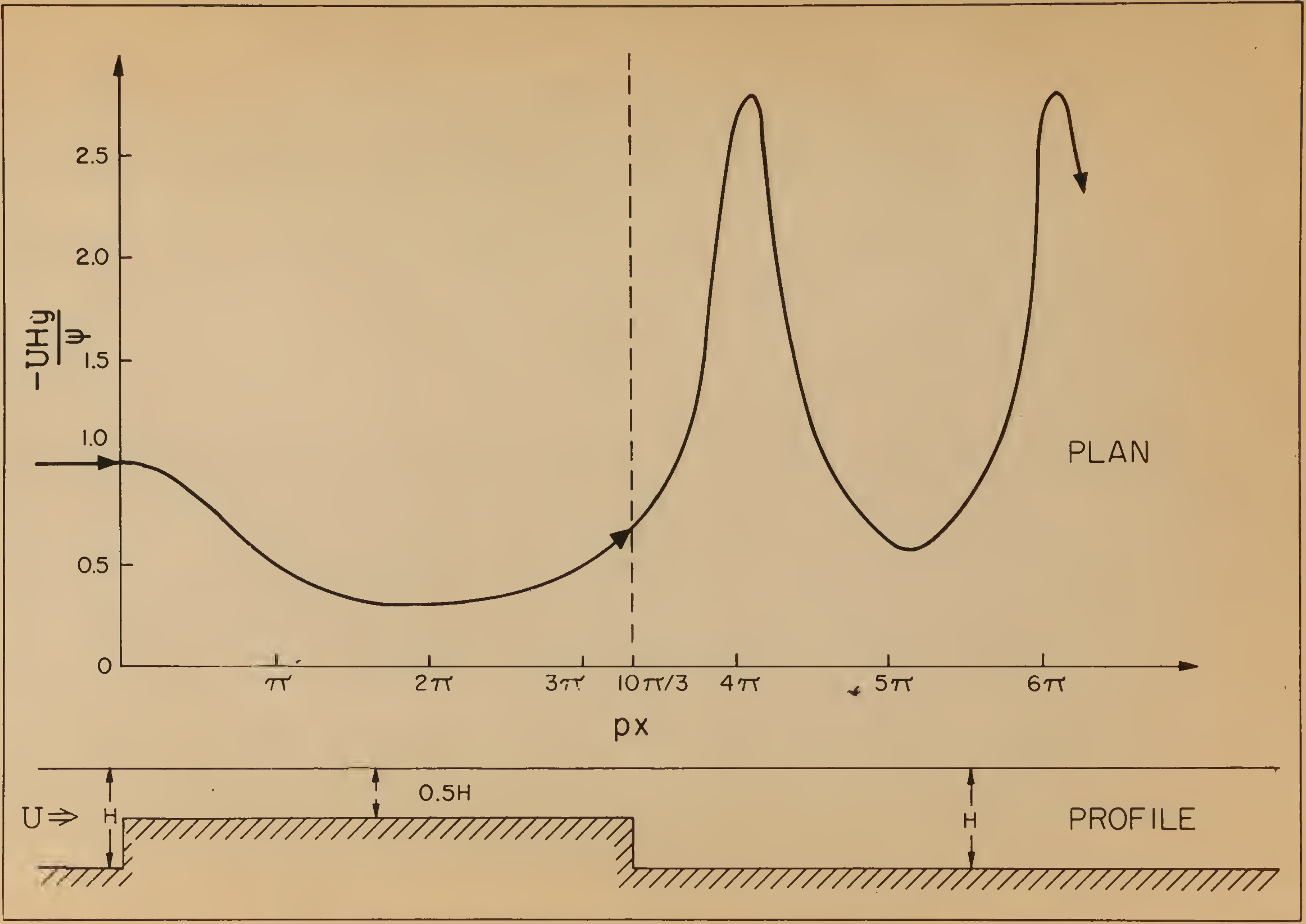

Figure 5. Zonal flow over a wide plateau

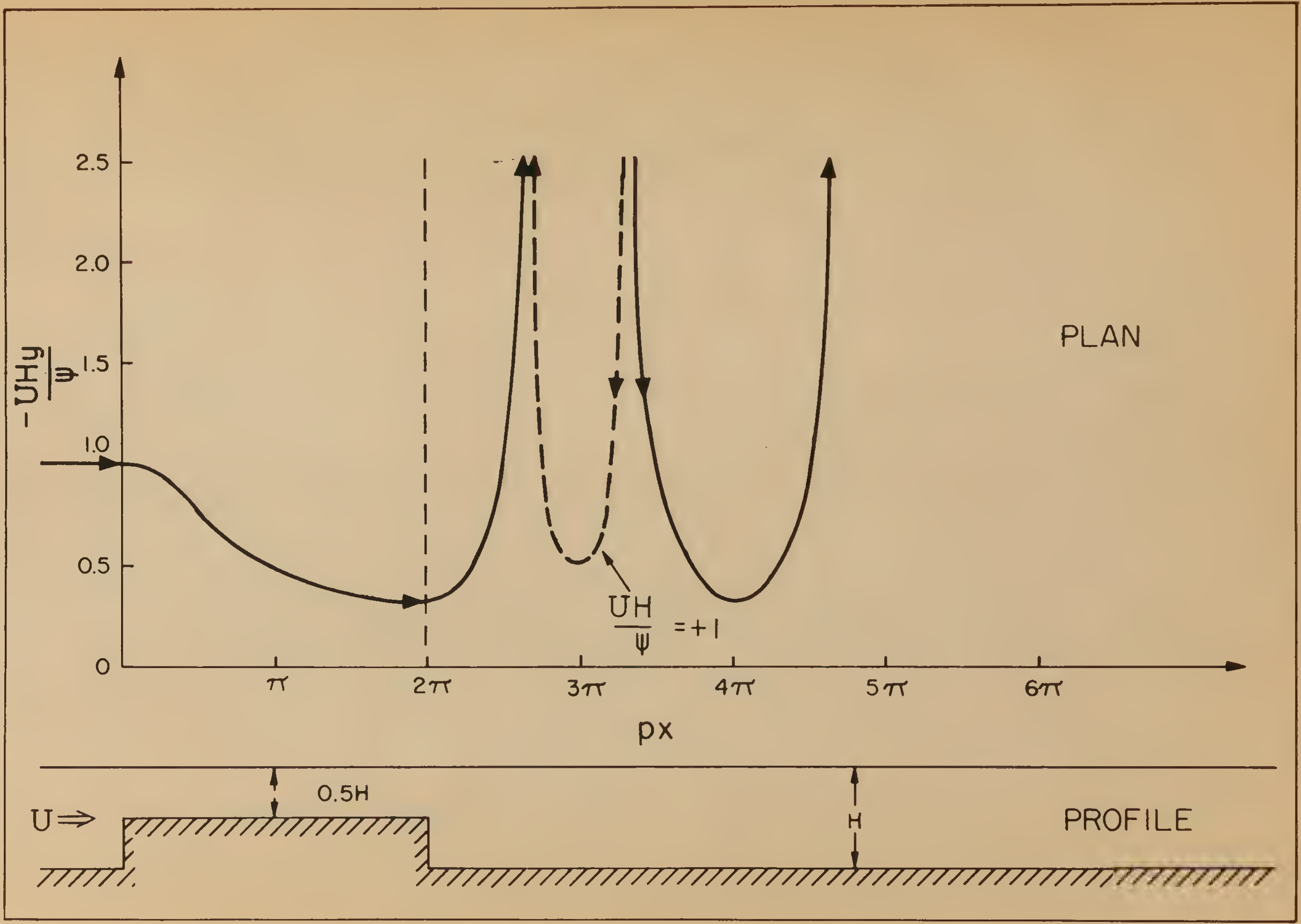

Figure 6. Unstable zonal flow over a plateau

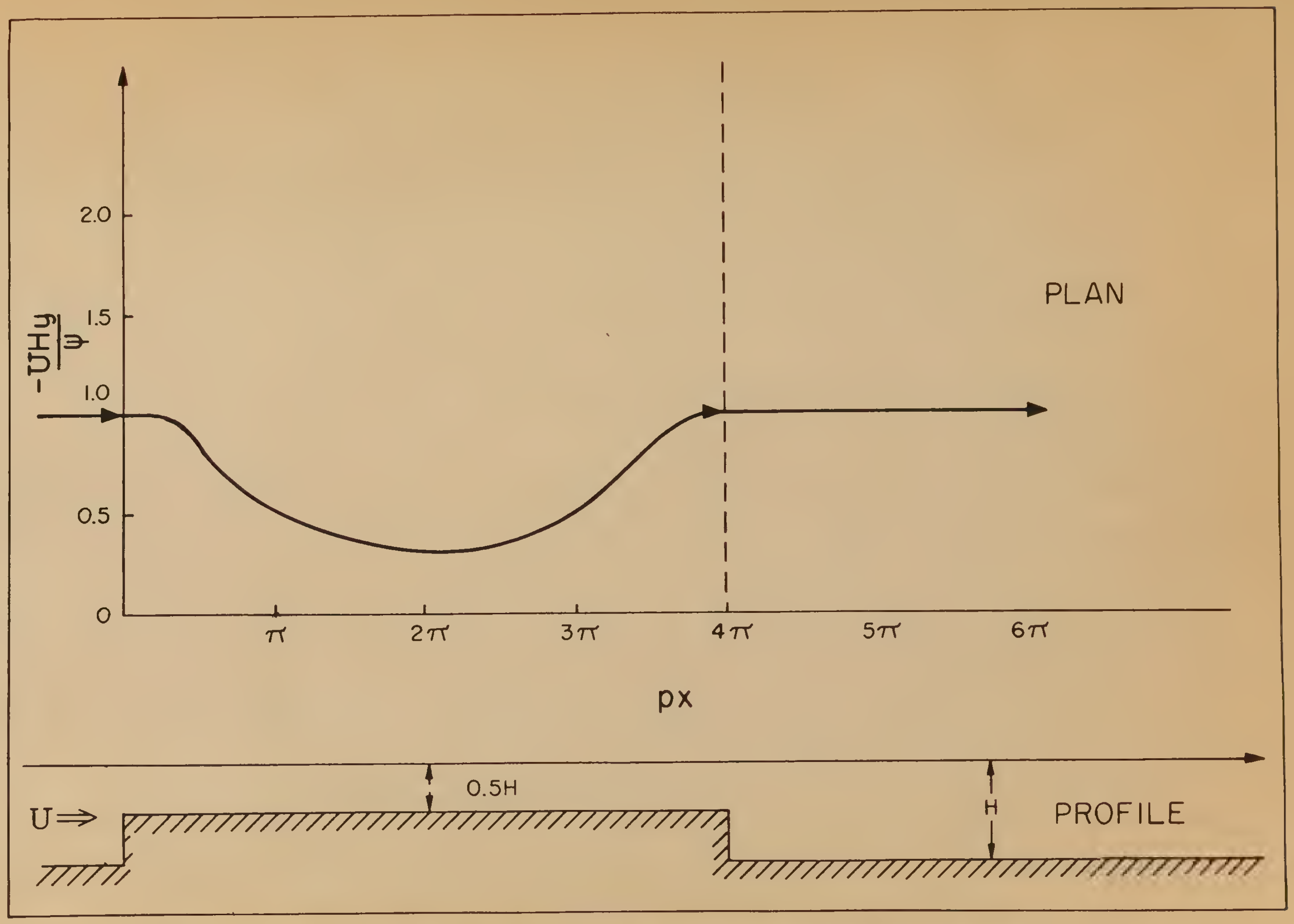

Figure 7. Zonal flow over a plateau (width: 1 wavelength)

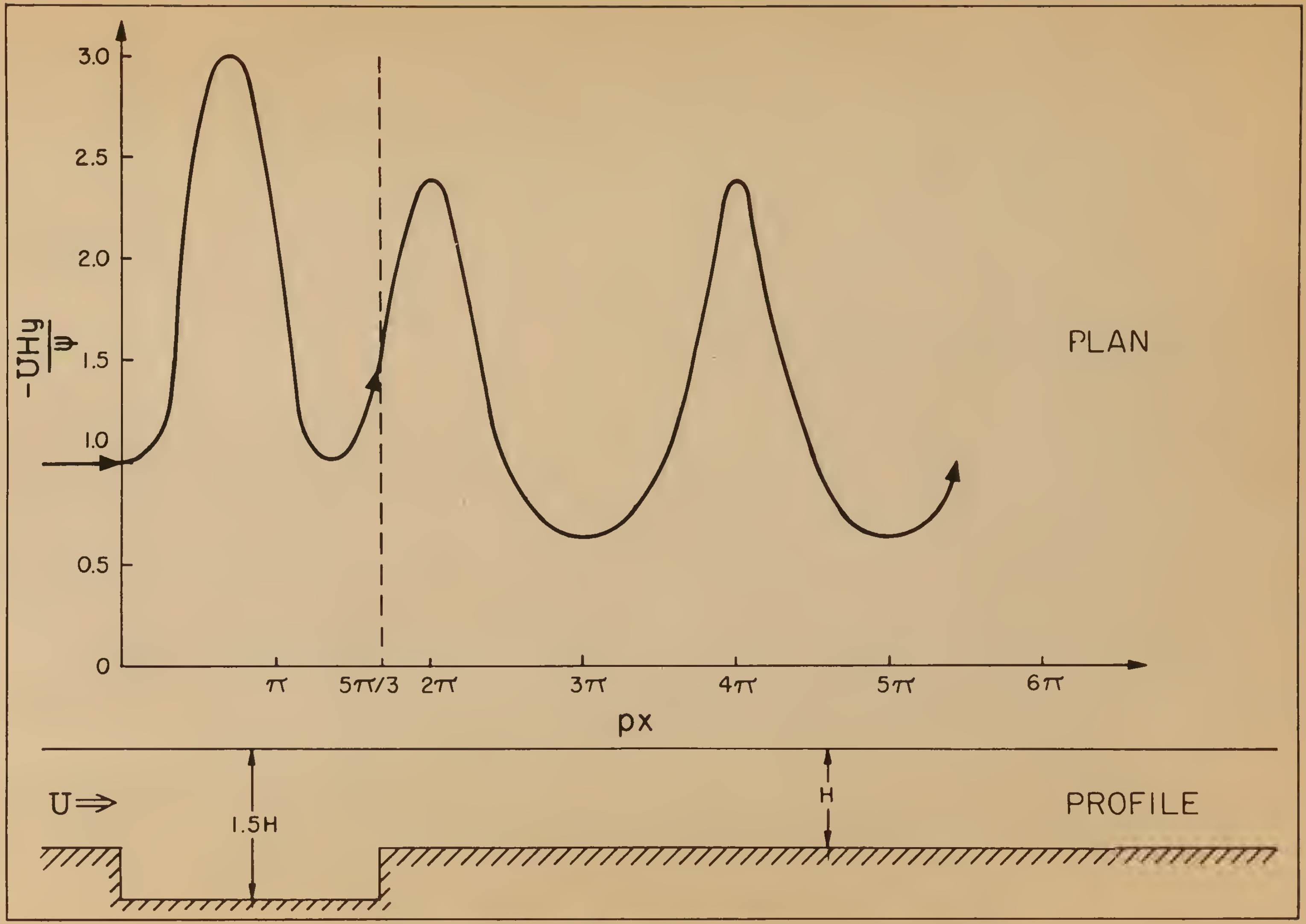

Figure 8. Zonal flow over an idealized valley

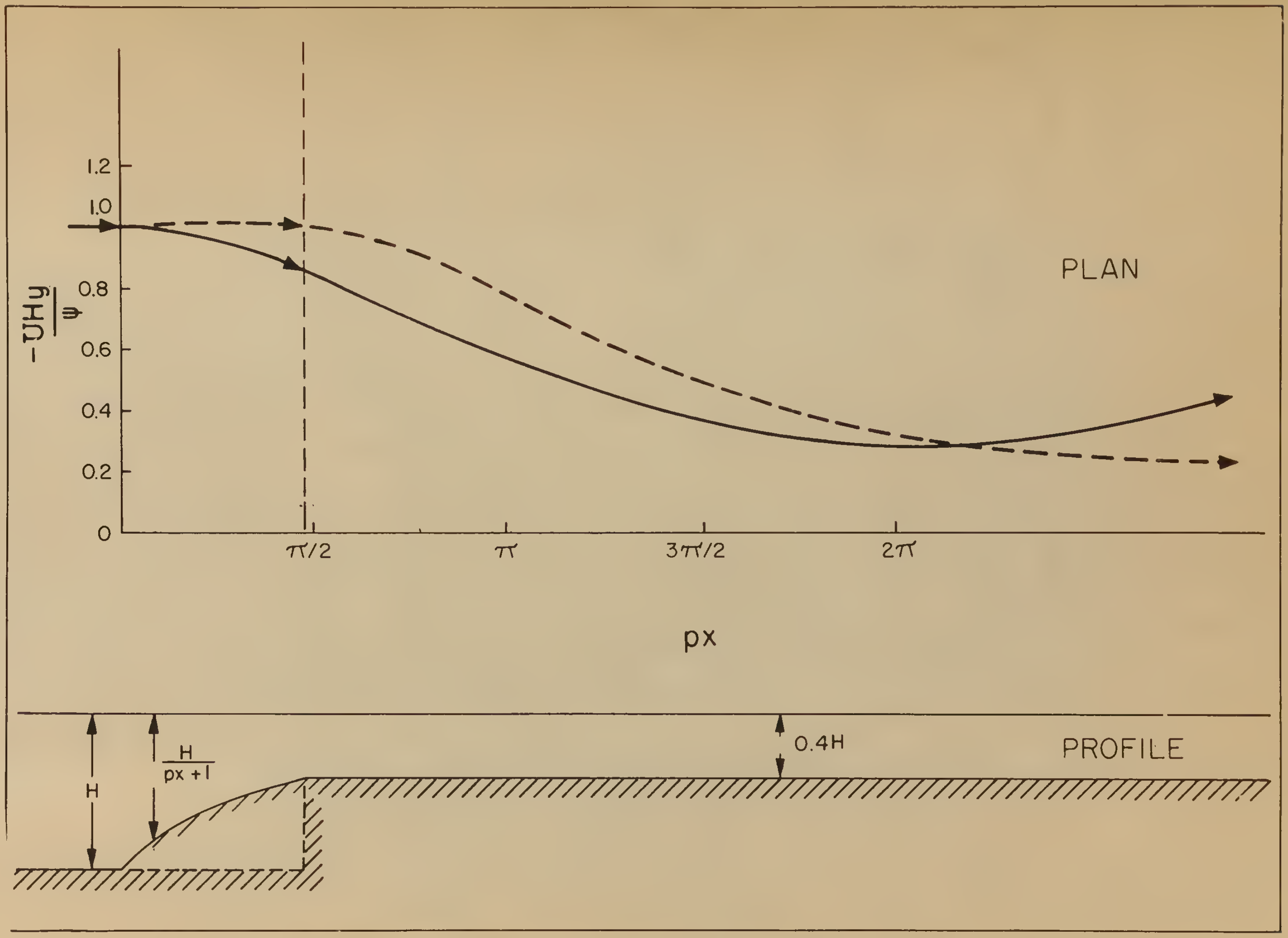

Figure 9. Zonal flow over a transition slope

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